Collective Labor Supply and Housework with Non-Participation of Women in Paid Labor

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Abstract

We estimate a collective time allocation model, where two-earner households behave as if the spouses maximize a household utility function, and where one-earner households, where only the man works, behave as if the spouses maximize a household utility function, conditional on the zero job-hour choice of the woman.

We find that the shape of the individual indifference curves are mainly influenced by leisure and the household income. For one-earner households, also household production is important and this is because there are relatively more children in these households. Differences between one-earner and two-earner households seem to reflect the difference in specialization behavior of the spouses.

Women in one-earner households have more bargaining power than their partner, and we find the opposite for two-earner households. The bargaining position in two-earner households is determined by the individual wages, while for one-earner households, it is determined by the wage rate of the man, the number of children and age.

Finally, we evaluate how one extra hour of female labor supply influences the household and the individual utility levels, assuming that the labor supply of women may be non-optimal for both one- and two-earner households. An increase of the woman’s labor hours would be a Pareto improvement for two-earner households. For one-earner households, we find that an extra hour of labor is beneficial for the household and for the woman, but not for the man.

JEL Codes: D12, D13, J22

Keywords: Collective household models; Household behavior; Labor supply; Intra-household; Time allocation
1 Introduction

Household labor supply is often examined for two-earner households, because the individual wages, as well as the positive number of labor hours can be observed for both spouses of these households. However, not all persons participate in paid labor. The non-participation decision may be different for different household types, and it may depend on the level of the individual wages. Thereby, the non-participation decision itself is endogenous with the number of paid labor hours and the exclusion of households, where household members do not participate in paid labor, could potentially result in a selectivity bias.

In this study, we examine how spouses allocate their time to leisure, paid labor and housework, and allow for the possibility that one of the partners in the household is not active in a paid job. As in our database of one-earner households, the male is, in most cases, the only income earner, we assume in the exposition that it will be the male partner who performs paid labor. Since, for the non-working female, we observe zero labor hours, it raises the question what information is conveyed by this observation? It may be that female unemployment is the result of an optimizing decision, i.e. we have voluntary unemployment. It may also be that unemployment is involuntary. Unfortunately, we do not have information in our data set on whether unemployment is voluntary or involuntary. As we know that much unemployment is involuntary, we cannot assume that zero labor hours corresponds to a situation of equilibrium.

We assume that spouses allocate their time to paid labor, leisure and housework and the decisions of these spouses concerning consumption, expenditures, and the allocation of time to paid labor, leisure and housework are interdependent. To allow for this interdependency, we make use of the collective model of household behavior. The collective model of household behavior has gained popularity because of its appealing properties\(^1\), however, empirical applications that make use of the collective model of household behavior and that take into account the non-participation of household members are still scarce\(^2\).

The collective model of household behavior has several advantages to its main competitors: the unitary model and the cooperative bargaining model. The advantage of the collective model to the cooperative bargaining model is that the predicted outcome of the household decision process is not based on a game theoretical equilibrium concept. Instead, the main assumption is that the outcome of a household decision process should yield a Pareto-efficient outcome, and no assumptions are made about the household decision process itself. Because the collective model allows for all Pareto-efficient outcomes, it holds that each efficient outcome predicted by a cooperative bargaining model can also be predicted by the collective model. The unitary model views each household as one decision unit (one person) with its own utility function and preferences. Spouses maximize the household utility function, and the efficient outcomes of the household

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\(^2\)Exceptions are Bloemen (2004), Blundell et al. (2005) and Vermeulen (2006)
decision process are driven entirely by comparative advantages. Because household members maximize the same a household utility function and individual preferences are not considered, nothing can be said on the intra-household allocation of welfare, which is clearly a drawback.\(^3\)

In the collective setting, the household decision process is described as if the household maximizes a weighted sum of the individual utility functions. These individual utility functions represent the preferences of the household members. The utility weights represent the division of bargaining power between the household members, and these weights are positive and add up to 1. As is clearly pointed out by Browning, Chiappori and Lechene (2006), these utility weights should depend on prices of consumption goods and/or wage rates, or else the model will collapse into a unitary model. We note that the core assumption of the collective model is that household decisions are Pareto efficient, which means that it is impossible to make one of the household members better-off without making another household member worse-off.

Since we do not have information on the expenditures of the family members, we formulate a public good version of the collective model, where household behavior is described as if the household chooses an optimal time allocation ‘bundle’. This is convenient, as there usually is data on time use, wage rates, distribution factors and preference factors. In particular, consumption goods are assumed to be public within the household. We also include household production by considering the time that spouses devote to housework and assuming for household production goods that they are public goods within the household. As is shown by Chiappori and Ekeland (2006), the individual preferences of this model are identifiable by assuming that spouses’ leisure is a private good. Therefore, each spouse’s leisure is assumed to be a private good, i.e. the husband does not benefit from the wife’s leisure, and conversely.

We assume that two-earner households behave as if the spouses maximize a household utility function, and where one-earner households, where only the man works, behave as if the spouses maximize a household utility function, conditional on the zero job-hour choice of the woman. It implies that the non-participation decision may not be optimal for the household as the zero job-hour choice may not optimal, but forced upon the female by external circumstances. In addition, we define a model where the labor supply decision may also be non-optimal for two-earner households.

By estimating the various models, we obtain more insights into how individual preferences and bargaining between spouses differ between one- and two-earner households. Furthermore, we obtain information on how individual utility and household utility are influenced by working an extra hour of paid labor. This is interesting as it gives more information about why women do or do not participate.

By choosing an empirical approach where we treat men’s labor supply as endogenous, where we consider housework and household production in the model, and where we consider the non-participation decision

\(^3\)In addition, the unitary model restricts household behavior by assuming that the household income is pooled and that marginal compensated wage changes of the spouses have a similar effect on each other labor supply. These restrictions have been empirically tested and are almost always rejected in the empirical literature. The collective model do not impose these unitary restrictions.
of women, we aim to contribute to a better understanding of the household decision process and the intra-
household bargaining process between the household members.

For this purpose, we use the British Household Panel Survey (BHPS), and distinguish between a sub-
sample of two-earner households and a sub-sample of one-earner households. The sub-sample of one-earner 
households consists of households where only the man works. As there are only 40 cases where only the 
woman works, we do not consider these households.

The structure of this paper is as follows. In Section 2, we discuss the theory. In Section 3, we describe 
the estimation method. In Section 4, the estimation results are presented and discussed. In Section 4.2, we 
discuss the estimation results, assuming that two-earner households, like one-earner households, behave as 
if they maximize a household utility function conditional on the constant paid labor hours of the woman. 
In Section 6 we simulate how utility is affected when the woman provides one more hours of paid labor. In 
Section 7 we summarize the main conclusions.

2 Theory

We start by considering two-earner households. Then we look for the modifications needed to describe the 
behavior of one-earner households.

Consider a two-earner household, where the preferences of spouse \( s \) \((s = m, f)\) are represented by the 
following direct utility functions:

\[
U_s = v_s(C, H, le_s, wh_s, jh_s) \tag{1}
\]

where \( v_s(\cdot) \) is twice continuously differentiable and strictly concave. The individual utility functions 
depend on the household consumption, \( C \), and the household production, \( H \). The utility function of spouse \( s \) 
depends on the time that is spent on leisure \( (le_s) \), housework \( (wh_s) \) and paid job hours \( (jh_s) \). It is somewhat 
unusual to include job hours directly in the utility function, as it is usually assumed that the working effort 
influences utility only negatively through a corresponding loss of leisure hours. However, there are numerous 
studies on life satisfaction that suggest that it is the experience of unemployment itself, rather than the loss 
of income through unemployment, that reduces life satisfaction (see Booth and van Ours (2007)). We also 
assume that men and women may derive direct utility from the performance of housework.

As we do not observe the household expenditures, we assume that consumption, \( C \), can be seen as a 
Hicksian composite good. This composite good represents total household consumption, the money value of 
which is household income, \( y \).\(^4\) Hence, in our model, \( C \) can be expressed as:

\[
C = y = w_m jh_m + w_f jh_f + y_u \tag{2}
\]

\(^4\)For simplicity, and without loss of generality, we assume that the price of this Hicksian composite good equals 1.
where $w_s$ stands for the wage rate of spouse $s$; and $y_u$ stands for unearned income of the household. Household production is generally not observed in data sets either. Therefore, the household production is represented by the household technology $h(wh_m, wh_f)$ that is a function of the hours that both spouses spend on housework. We assume the following functional form for this household technology:

$$H = h(wh_m, wh_f) = wh_m + \gamma wh_f$$

(3)

where $\gamma$ represents the marginal productivity of the woman relative to that of the man.

We model the household consumption and the household production as if they are public goods in the household. As the aggregated level of household income (that represents consumption) and the weighted sum of the individual hours spent on housework each represent one value for each household, it is not possible to examine how the various goods are distributed over the household members.

According to the Collective Model (CM), the household decision process can be described as if the household maximizes a weighted sum of the individual utility functions, subject to the individual time constraints that spouses face. Spouses are assumed to choose the optimal bundle $(le_m, wh_m, le_f, wh_f)$ that maximizes the following household utility function:

$$U_h = \pi(w_m, w_f, d) \cdot v_m(y, H, le_m, wh_m, T - le_m - wh_m)$$

$$+ (1 - \pi(w_m, w_f, d)) \cdot v_f(y, H, le_f, wh_f, T - le_f - wh_f)$$

(4)

where $H$ is defined as in (3), and $y$ is redefined as:

$$y = w_m(T - le_m - wh_m) + w_f(T - le_f - wh_f) + y_u$$

(5)

where $w_m$ and $w_f$ stand for the hourly wage rates. The number of job hours of spouse $s$ ($jh_s$) that appears in (5) and directly in the individual utility function is rewritten, using the individual time constraint, as $(T - le_s - wh_s)$, where $T$ is the total time endowment per week. By substitution, we have incorporated the household budget constraint and the individual time constraints in the model.

The individual utility functions are weighted by the utility weight $\pi(\cdot)$ that is assumed to lie in the interval $[0,1]$. An intuitive interpretation of this weight is that it represents the division of bargaining power between the spouses. In this study, the function $\pi(\cdot)$ depends on the individual wage rates, but also depends on several distribution factors ($d$). These factors influence the division of bargaining power between the spouses, but not the preferences of the spouses. Browning et al. (2006) point out that, although generally $\pi$ is considered as a function, there has been confusion about what the arguments of this function should be. They show that, when $\pi$ is ‘misspecified’ and does not depend on the individual wage rates, the CM collapses into the conventional unitary model.
According to the unitary model spouses behave as if they maximize the same household utility function, and the efficient outcomes of the household decision process are driven entirely by comparative advantages. It follows that nothing can be said on the intra-household allocation of welfare. In addition the unitary model restricts household behavior by assuming that the household income is pooled and that marginal compensated wage changes of the spouses have a similar effect on each other labor supply. The latter is also known as the Slutsky symmetry assumption. The restrictions of the unitary model are empirically tested and almost always rejected in the empirical literature (see, among others, Ashworth and Ulph (1981), Kooreman and Kapteyn (1986), Thomas (1990), Browning and Costas (1991), Browning, F, Bourguignon, Chiappori and Lechene (1994), Kawaguchi (1994), Fortin and Lacroix (1997), Lundberg, Pollak and Wales (1997), Browning and Chiappori (1998), Ward-Batts (2002)).

Since consumption goods and household production goods are assumed to be public, we refer to this model as a Public Good Version of the Collective Model (PGVCM). For identification purposes, each spouse’s leisure is assumed to be a private good, i.e. the husband does not benefit from the wife’s leisure, and conversely (see Chiappori and Ekeland (2006)).

The corresponding system of first-order-conditions (FOCs) with respect to the man’s and woman’s leisure and housework is then:

\[
\begin{align*}
\frac{\partial U_h}{\partial le_m} &= \pi \frac{\partial U_m}{\partial le_m} + (1 - \pi) \frac{\partial U_f}{\partial le_m} \\
\frac{\partial U_h}{\partial wh_m} &= \pi \frac{\partial U_m}{\partial wh_m} + (1 - \pi) \frac{\partial U_f}{\partial wh_m} \\
\frac{\partial U_h}{\partial le_f} &= \pi \frac{\partial U_m}{\partial le_f} + (1 - \pi) \frac{\partial U_f}{\partial le_f} \\
\frac{\partial U_h}{\partial wh_f} &= \pi \frac{\partial U_m}{\partial wh_f} + (1 - \pi) \frac{\partial U_f}{\partial wh_f}
\end{align*}
\]

(6)

Let us focus on the first partial derivative with respect to male leisure. This partial derivative consists of two terms. The first term represents the male part of the collective utility function, while the second term represents the female part of the collective utility function. It follows that the leisure choice of the man influences the household utility through the utility of the man and the utility of the woman, and vice versa.

In order to see how this happens, we can write the first FOC more extensively as:

\[
\frac{\partial U_h}{\partial le_m} = \pi \left[ \frac{\partial U_m}{\partial le_m} + \frac{\partial U_m}{\partial y} \frac{\partial y}{\partial le_m} + \frac{\partial U_m}{\partial jh_m} \frac{\partial jh_m}{\partial le_m} \right] + (1 - \pi) \left[ \frac{\partial U_f}{\partial y} \frac{\partial y}{\partial le_m} \right]
\]

(7)

From the partial derivative in (7), we can deduce through which components the household utility function is influenced. The first term (\(\frac{\partial U_m}{\partial le_m}\)) indicates that the man’s leisure influences the household utility directly through the utility function of the male. This is a consequence of the identifying assumption that the individual leisure is a private good.
Because we replace job hours by the individual time constraints, the leisure of the man influences the household utility through consumption $y$ and through the man’s job hours. As consumption goods are public, the household utility is influenced by the leisure time of the man, through the utility function of both man and woman. As both utility functions are differently weighted in the household utility function, the sum of the individual partial effects are weighted by the utility weight $\pi$ as well.

We do not repeat this exercise for the other FOCs, as the intuition is the same. Solving the FOCs for the choice variables leisure and housework (and consequently job hours) gives the following system of demand functions:

\[ z = f(w_m, w_f, y_u, d) \]  
\[ (8) \]

where we introduce the shorthand notation $z$ that stands for the solution vector $z = (le_m, wh_m, le_f, wh_f)$. These ‘time’ demand functions are functions of the wage rates, the unearned income and the distribution factors that appear in the utility weight.

The model above applies for two-earner households. However, we have now to consider the situation where the female partner is not participating in the labor market.

For these one-earner households, we assume that the household behavior can be described as if the household maximizes a household utility function that is conditional on the zero job-hour choice of the woman. Optimization is performed only with respect to the three variables $(le_m, wh_m, le_f)$. Hence equation (4) is rewritten such that, for these one-earner households, spouses choose the optimal bundle $(le_m, wh_m, le_f)$ that maximizes the household utility function:

\[ U_h = \pi(w_m, \bar{w}_f, d) \cdot v_m(C, H, le_m, wh_m, T - le_m - wh_m) + (1 - \pi(w_m, \bar{w}_f, d)) \cdot v_f(C, H, le_f, T - le_f) \]  
\[ (9) \]

where $H$ and $C$ are defined as:

\[ C = y = w_m(T - le_m - wh_m) + y_u \]
\[ H = wh_m + \gamma(T - le_f) \]  
\[ (10) \]

To simplify the notation, we leave out the subscript, indicating that the situation applies for one-earner households only. Household consumption is again considered as a Hicksian composite good, and therefore we can represent the total household consumption $C$ by the household income $y$.

\[ \text{Because we assume additive individual utility functions, i.e. } v_f(C, H, le_f, wh_f, jh_f) = v_f^A(C, H, le_f, wh_f) + v_f^B(jh_f), \text{ we can exclude the zero job hours of the woman as it does not affect the optimal solution.} \]
When we compare equation (9) with equation (4), the changes are the following. As the woman is not participating in the labor market, she works zero job hours, but we have no information suggesting that an equilibrium condition is fulfilled with respect to \( jh_f \). We do not observe an hourly wage rate for her, which we can use as a determinant for the distribution of power between male and female. Her economic power is reflected by \( \tilde{w}_f \), which is the hourly wage that she would receive if she decided to work a positive number of labor hours. This market wage \( \tilde{w}_f \) will have to be estimated.

The utility function of the household is now conditional on the zero labor supply choice of the woman. When the woman does not work, she may receive social benefits that influences her behavior. The utility function of the male essentially remains the same, but the non-participation of the woman affects his utility through household income and the household production of goods.

As for the two-earner case, we substitute the individual time constraints in the utility functions and the budget constraint of the household. As the time constraint of the woman is \( wh_f = T - le_f \), we need to solve the the \( FOCs \) with respect to the choice variables \( le_m, le_f \) and \( wh_m \). The corresponding system of first-order-conditions, by which we can determine the optimal time allocation for one-earner households is then

\[
\begin{align*}
\frac{\partial U_h}{\partial le_m} &= \pi \frac{\partial U_m}{\partial le_m} + (1 - \pi) \frac{\partial U_f}{\partial le_m}, \\
\frac{\partial U_h}{\partial wh_m} &= \pi \frac{\partial U_m}{\partial wh_m} + (1 - \pi) \frac{\partial U_f}{\partial wh_m}, \\
\frac{\partial U_h}{\partial le_f} &= \pi \frac{\partial U_m}{\partial le_f} + (1 - \pi) \frac{\partial U_f}{\partial le_f},
\end{align*}
\]

(11)

where there are only three parameter-identifying conditions instead of four in the case of the two-earner household.

Focusing on the third \( FOC \) and writing this \( FOC \) more extensively gives:

\[
\frac{\partial U_h}{\partial le_f} = \pi \left[ \frac{\partial U_m}{\partial H} \frac{\partial H}{\partial le_f} \right] + (1 - \pi) \left[ \frac{\partial U_f}{\partial le_f} + \frac{\partial U_f}{\partial wh_f} \frac{\partial wh_f}{\partial le_f} + \frac{\partial U_f}{\partial H} \frac{\partial H}{\partial le_f} \right]
\]

(12)

Besides the direct effect of leisure on household utility, the amount of leisure influences household utility indirectly through housework and the household production. This is because the time constraint of the woman is substituted in her utility.

When we solve the \( FOCs \) for the choice variables \( le_m, le_f \) and \( wh_m \), we obtain the following system of demand functions:

\[
\tilde{z} = g(w_m, \tilde{w}_f, y_u, d)
\]

(13)

where \( \tilde{z} \) stands for the solution vector for one-earner households, i.e. \( \tilde{z} = (le_m, wh_m, le_f) \). This vector contains one element less than the solution vector we found for the two-earner household. The time-demand
functions depend on the wage rate of the man, the predicted market wage rate of the woman, the unearned income and the distribution factors that appear in the utility weight.

The model for one-earner households assumes that the man’s job hours and the amount of leisure and housework of both spouses are optimally chosen. We do not know whether the zero job hours of the female is the result of optimization or whether it is forced upon her by the labor market. However, the fact that we do not have all the information we would like to have does not prevent us from identifying and estimating a subset of preference parameters.

3 The Empirical Model and Estimation Method

For the estimation procedure we distinguish between two-earner households and one-earner households, where only the man works. First, we focus on the two-earner households, and afterwards we consider how the empirical model changes for one-earner households.

Two-earner households

The preferences of spouse \( s (s = m, f) \) are represented by the following log-additive utility functions:

\[
v_s(y, H, le_s, wh_s, jh_s) = \alpha_{s,1} \ln(le_s) + \alpha_{s,2} \ln(wh_s) + \\
[\alpha_{s,3} + \alpha_{s,3I} \ln(fs + 1)] \ln(H) + \alpha_{s,4} \ln(y) + \alpha_{s,5} \ln(jh_s)
\]  

(14)

where again the household consumption, \( C \), is denoted as:

\[
C = y = w_m \cdot jh_m + w_f \cdot jh_f + y_u
\]

(15)

The weekly net wage rates and the unearned income are measured in UK pounds for both spouses. As in Section 2, the household production is defined as

\[
H = wh_m + \gamma \cdot wh_f
\]

(16)

In (14), the utility effect of the total household production depends on family size. In this way we recognize that the utility derived from the total (weighted) housework hours \( (H) \) is likely to be influenced by the size of the family. This dependency is modeled by including an interaction term between family size \( fs \) and \( H \).
Furthermore, the following time constraints apply:

\[ jh_s = T - le_s - wh_s \]
\[ 0 \leq le_s \leq T \]
\[ 0 \leq wh_s \leq T \]  \hspace{1cm} (17)

When we substitute (15) and (16) into the utility function described in (14), the behavior of household \( n \) can be described as if the household maximizes a household utility function of the following type:

\[
\max_{le_n,m,wh_n,m,le_n,f,wh_n,f} U_{n,h} = \pi_n v_{n,m} + (1 - \pi_n) v_{n,f} 
\]

subject to (17) with

\[
v_s = \alpha_s,1 \ln(le_s) + \alpha_s,2 \ln(wh_s) + [\alpha_s,3 + \alpha_s,3_l \ln(f_s + 1)] \ln(wh_m + \gamma wh_f) 
+ \alpha_{s,4} \ln(w_m jh_m + w_f jh_f + y_u) + \alpha_{s,5} \ln(jh_s)  
\]

It is important to recognize that job hours of both spouses are replaced by \( T - le_s - wh_s \), such that there are four choice variables, i.e. the time spent on leisure and housework by both spouses.

For now, we assume that the utility weight is constant for each household, although it varies over the households. The solution to the optimization problem for each household can be obtained by deriving the system of first-order conditions:

\[
\frac{\partial U_h}{\partial le_m} = \pi[p \frac{\alpha_{m,1}}{le_m} - \frac{\alpha_{m,4} w_m}{y} - \frac{\alpha_{m,5}}{jh_m}] + (1 - \pi)\frac{-\alpha_{f,4} w_m}{y} 
\]
\[
\frac{\partial U_h}{\partial wh_m} = \pi[p \frac{\alpha_{m,2}}{wh_m} + \frac{\alpha_{m,3}}{H} + \frac{\alpha_{m,3_l} \log(f_s)}{H} - \frac{\alpha_{m,4} w_m}{y} - \frac{\alpha_{m,5}}{jh_m}] 
+ (1 - \pi)[\frac{\alpha_{f,3}}{H} + \frac{\alpha_{f,3_l} \log(f_s)}{H} - \frac{\alpha_{f,4} w_m}{y}]  
\]
\[
\frac{\partial U_h}{\partial le_f} = \pi[-\frac{\alpha_{m,4} w_f}{y} + (1 - \pi)\frac{\alpha_{f,1}}{le_f} - \frac{\alpha_{f,4} w_f}{y} - \frac{\alpha_{f,5}}{jh_f}] 
\]
\[
\frac{\partial U_h}{\partial wh_f} = \pi[p \frac{\alpha_{m,3}}{H} + \frac{\alpha_{m,3_l} \log(f_s)}{H} - \frac{\alpha_{m,4} w_f}{y}] 
+ (1 - \pi)[\frac{\alpha_{f,2}}{wh_f} + \frac{\alpha_{f,3}}{H} + \frac{\alpha_{f,3_l} \log(f_s)}{H} - \frac{\alpha_{f,4} w_f}{y} - \frac{\alpha_{f,5}}{jh_f}]  
\]

To obtain the optimal solution for a two-earner household, we would normally set the partial derivatives in (19) to zero and solve the system for the endogenous choice variables leisure and housework. However, these functions are highly non-linear in the estimation parameters, and therefore we use the more convenient two-step iterative estimation procedure to estimate the unknown parameter estimates. This estimation procedure
relates to the Wald test criterion approach (see Wales and Woodland (1983) and Blundell and Robin (1999)). Before we can explain, and apply the estimation method, we simplify the system in (19).

We rewrite the first partial derivative, \( \frac{\partial U_h}{\partial e_m} \), in a more convenient way, so that each model parameter \( \alpha \) has one accompanying coefficient \( x \):

\[
\frac{\partial U_h}{\partial e_m} = \pi [\alpha_{m,1}x_{1,m,1} + \alpha_{m,4}x_{1,m,4} + \alpha_{m,5}x_{1,m,5}] + (1 - \pi)\alpha_{f,4}x_{1,f,4}
\]

Equation (20) is essentially the same as equation (7), where the partial derivative with respect to man’s leisure was extensively rewritten. Leisure influences the household utility through the utility function of the man in three ways: (1) directly \( (x_{1,m,1}) \); (2) through the household income \( (x_{1,m,4}) \); and (3) through job hours \( (x_{1,m,5}) \). Household utility is also influenced by household income through the utility of the woman \( (x_{1,f,4}) \). Similarly, we rewrite the other three FOCs as well and we show the full system of FOCs in Appendix 5A.

The linear system of FOCs can be written as:

\[
\begin{bmatrix}
\pi x'_{1m} & (1 - \pi)x'_{1f} \\
\pi x'_{2m} & (1 - \pi)x'_{2f} \\
\pi x'_{3m} & (1 - \pi)x'_{3f} \\
\pi x'_{4m} & (1 - \pi)x'_{4f}
\end{bmatrix}
\begin{bmatrix}
\alpha_m \\
\alpha_f
\end{bmatrix}
= \pi X'_{m} (1 - \pi)X'_{f}
\begin{bmatrix}
\alpha_m \\
\alpha_f
\end{bmatrix}
= 0
\]  

(21)

where the index 1 refers to the \( x \)-vector in the first FOC and where \( X'_{m} \) and \( X'_{f} \) are \( (4 \times 6) \)-matrices. The matrix \( X'_{i} \) contains some zero elements. For example, the parameter \( \alpha_{m,2} \) does not appear in equation (20) and the coefficient that belongs to this parameter in matrix \( X'_{m} \) is therefore set to zero.

More concisely, we can now define a \( (4 \times 12) \)-matrix \( X'_{n} \) by:

\[
[\pi_n X'_{n,m} \quad (1 - \pi_n)X'_{n,f}] = X'_{n}
\]

(22)

The gradient of the household utility function \( U_h(z) \) can then be written as \( X'_{n} \alpha \), where \( z \) stands for the solution vector \( z = (le_m, wh_m, le_f, wh_f) \).

The utility weight is a function that must depend on wage rates and usually depends on certain factors that are thought to influence the distribution of bargaining power. We model the utility weight as:

\[
\pi_n(w_m, w_f, d) = N(\beta_m \log(w_{n,m}) + \beta_f \log(w_{n,f}) + \sum_{j=3}^{J} \beta_j \cdot d_{j,n})
\]

(23)

where \( N(\cdot) \) stands for the standard normal distribution function. The use of a normal distribution function is convenient since \( \pi_n(w_m, w_f, d) \in (0, 1) \), while the \( \beta \)-parameters are not bounded and may take any real
value. The utility function of the man is weighted more heavily in the collective utility function when $\pi$ increases and this is at the expense of the utility of the woman. We would expect that $w_m$ affects the utility weight positively as a wage increase of the man is likely to cause a change in the utility weight that is to his advantage, and that the coefficient of $w_f$ will be negative. The distribution factors in our model are unearned income, the number of children, and the age of both spouses. We note that the inclusion of a constant would allow for the situation where one partner’s utility is structurally overweighted. However, we found that the estimate of this constant was consistently insignificant, and therefore we do not include it in the distribution function. Other distribution factors, such as differences in educational level and age level, were also consistently insignificant, and so we also omitted these factors from the model.

**One-earner households: Only the man works**

One earner households are assumed to maximize a household utility function with respect to housework $wh_m, wh_f$ and the male’s leisure $le_m$, while $jh_f$ is fixed at zero. For these households the time constraints are:

$$
\begin{align*}
    jh_m &= T - le_m - wh_m \\
    wh_f &= T - le_f \\
    jh_f &= 0 \\
    0 \leq le_s &\leq T \\
    0 \leq wh_s &\leq T
\end{align*}
$$

The time constraint of the woman is $T = le_f + wh_f$, and spouses choose the optimal time allocation. We notice that the zero labor hours of the woman may not be optimally chosen.

The behavior of one-earner households can be described as if the household maximizes the following household utility function:

$$
\max_{le_a, wh_a, le_f} U_{n,h} = \pi_n v_{n,m} + (1 - \pi_n) v_{n,f}
$$

with

$$
v_s = \alpha_{s,1} \ln(le_s) + \alpha_{s,2} \ln(wh_s) + [\alpha_{s,3} + \alpha_{s,3f} \ln(fs + 1)] \ln(wh_m + \gamma(T - le_f)) + \alpha_{s,4} \ln(w_m jh_m + y_s) + I \cdot \alpha_{s,5} \ln(jh_s)
$$

subject to (24). The indicator variable $I$ equals 1 in the utility function of the male and 0 in the utility function of the woman. The woman’s utility function does not contain the $\alpha_{f,5} \ln(jh_f)$-term when the woman does not perform paid work. The parameter $\alpha_{f,5}$ is not identified. For one-earner households, we therefore set $\alpha_{f,5}$ to zero. We note that we are not interested in the absolute values of $\alpha$, but only in their relative
values.

The spouses of one-earner households maximize with respect to the three choice variables \( le_{n,m}, wh_{n,m}, le_{n,f} \), yielding a system of three FOCs. The expressions for \( \frac{\partial U_h}{\partial le_m} \) and \( \frac{\partial U_h}{\partial wh_m} \) are similar for one- and two-earner households. Therefore, we show only the partial derivative with respect to the woman’s leisure:

\[
\frac{\partial U_h}{\partial le_f} = \pi \left( -\frac{\alpha_{m,3}\gamma}{H} - \frac{\alpha_{m,3l}\gamma \log(fs)}{H} \right) + (1 - \pi) \left( \frac{\alpha_{f,1}}{le_f} - \frac{\alpha_{f,2}}{wh_f} - \frac{\alpha_{f,3}\gamma}{H} - \frac{\alpha_{f,3l}\gamma \log(fs)}{H} \right) \tag{26}
\]

More concisely, we may rewrite (26):

\[
\frac{\partial U_h}{\partial le_f} = \pi [\alpha_{m,3} x_{3,m,3} + \alpha_{m,3l} x_{3,m,3l}] + (1 - \pi) [\alpha_{f,1} x_{3,f,1} + \alpha_{f,2} x_{3,f,2} + \alpha_{f,3} x_{3,f,3} + \alpha_{f,3l} x_{3,f,3l}] \tag{27}
\]

such that for one earner households the linear system of three equations becomes

\[
\begin{bmatrix}
\pi x'_{1m} (1 - \pi) x'_{1f} \\
\pi x'_{2m} (1 - \pi) x'_{2f} \\
\pi x'_{3m} (1 - \pi) x'_{3f}
\end{bmatrix}
\begin{bmatrix}
\alpha_m \\
\alpha_f
\end{bmatrix}
= \pi X'_m (1 - \pi) X'_f
\begin{bmatrix}
\alpha_m \\
\alpha_f
\end{bmatrix}
= 0 \tag{28}
\]

As in equation (21), the index 1 refers to the \( x \)-vector in the first FOC. However, the matrices \( X'_m \) and \( X'_f \) are now \((3 \times 6)\)- and \((3 \times 5)\)-matrices, and the parameter vector \( \alpha_f \) is now a five-vector, as the parameter \( \alpha_{f,5} \) is set to 0.

For one-earner households \( n \), we define a \((3 \times 11)\)-matrix \( X'_n \) by:

\[
\begin{bmatrix}
\pi x'_{1n,m} (1 - \pi) x'_{1n,f} \\
\pi x'_{2n,m} (1 - \pi) x'_{2n,f} \\
\pi x'_{3n,m} (1 - \pi) x'_{3n,f}
\end{bmatrix}
= X'_n
\tag{29}
\]

Again the left-hand-side of (21) is the gradient of the household utility function \( U_h(\tilde{z}) \), where \( \tilde{z} \) stands for the solution vector \( \tilde{z} = (le_m, wh_m, le_f) \) for one-earner households.

The definition of the utility weight function is similar for both one- and two-earner households, although we note that for the wage rate of the non-participating woman we use the predicted market wage rate, as we do not observe her wage rate.

To distinguish between one-earner and two-earner households we refer to the one-earner households by using the subscript \( o \), because only one of the spouses works. We refer to the two-earner households by using
the subscript $b$, because both spouses work. We can rewrite the system of FOCs in (22) and (30) as:

$$X'_{n,h} \alpha_h = 0, \text{ for } h = b, o \quad (30)$$

The Estimation Method

We divide the population $S$ into a sample of two-earner households ($S_b$), and a sample of one-earner households ($S_o$). For the sample of two-earner households we have the following system:

$$X'_{n,b} \alpha_b = 0, \quad \forall n \in S_b \quad (31)$$

Let us assume that the values for the $\beta$-parameters are known, and hence $\pi_n(\cdot)$ is known for each household. As the equality does not hold exactly, we assume for households in the sample $S_b$ that:

$$X'_{n,b} \alpha_b = \varepsilon_{n,b}, \quad \forall n \in S_b \quad (32)$$

where $\varepsilon$ is the stochastic error vector with $\varepsilon \sim N(0, \Sigma_\varepsilon)$ that is not correlated between households (i.e. $E(\varepsilon_n, \varepsilon_{n'}) = 0$ if $n \neq n'$).

By introducing the nuisance (4-)vector $y_n$, we can write equation (32) as

$$y_n = X'_{n,b} \alpha_b + \varepsilon_{n,b}, \quad \forall n \in S_b \quad (33)$$

When setting $y_n = 0$ for all $n$, the system in (33) can be estimated by minimizing $\sum_{n \in S_b} \varepsilon'_{n,b} \Sigma^{-1}_\varepsilon \varepsilon_{n,b}$ with respect to the $\alpha_b$-parameters, subject to the parameter constraints $\sum \alpha_{m,b} = 1$ and $\sum \alpha_{f,b} = 1$. This is equivalent to estimating the system in (33) with a Seemingly Unrelated Least Squares (SUR), where we exclude the ‘trivial’ solution, i.e. all $\alpha$ parameters are zero, by imposing that $\sum \alpha_{m,b} = 1$ and $\sum \alpha_{f,b} = 1$.

In order to estimate the model for one-earner households, we need to modify the estimation model slightly. When the woman does not participate in paid labor, her job hours are fixed at $j_{h_f} = 0$. Those households maximize a conditional utility function where they again maximize with respect to $l_{e_{n,m}}, w_{h_{n,m}}, l_{e_{n,f}}, w_{h_{n,f}}$, but where the time constraint for the woman is now $l_{e_{n,f}} + w_{h_{n,f}} = T$, as $j_{h_f} = 0$. We eliminate the variable $w_{h_{n,f}} = T - l_{e_{n,f}}$, and maximize with respect to the three remaining variables $l_{e_{n,m}}, w_{h_{n,m}}, l_{e_{n,f}}$, yielding three FOCs, again linear in the parameters $\alpha$. It follows that we get a 3-equations system:

$$y_n = X'_{n,o} \alpha_o + \varepsilon_{n,o}, \quad \forall n \in S_o \quad (34)$$
By minimizing the likelihood \( \sum_{n \in S_o} e_n' \Sigma^{-1} e_n \) under the assumptions that \( \sum \alpha_m = 1 \) and \( \sum \alpha_f = 1 \), and with \( \Sigma^{-1} \) being a (3x3)-matrix, we can retrieve the \( \alpha \)-parameters.

In the system in (34), the \( \alpha_{f,5} \)-parameter does not appear so that it cannot be estimated. We assume that, for one-earner households, \( \alpha_{f,5} \) is 0 and impose that the other \( \alpha \)-parameters for the woman add up to 1, so that the utility function of the woman is linearly homogeneous.

In section 4.2, we examine how the estimation results for two-earner households change, when we assume that these households maximize a conditional utility function as well. Those households then maximize with respect to \( le_{n,m}, wh_{n,m}, le_{n,f}, wh_{n,f} \), where the time constraint for the woman is \( le_{n,f} + wh_{n,f} + \tilde{j}h_{n,f} = T \). The constant \( \tilde{j}h_{n,f} \), represents the number of hours that the woman participates in paid labor. As is the case for one-earner households, we obtain a 3-equations system that can be estimated with a SUR method under the assumption of linear homogenous utility functions.

When we estimate the 3-equations system separately for one- and two-earner households, we assume both household types have different parameter vectors \( \alpha \). Besides estimating the 3-equations system separately for one- and two-earner households, it is also possible to estimate this model jointly for both household types. In this case we assume that both household types have the same parameter values \( \alpha \). Under this assumption, we minimize the sum of squared residuals \( \sum_{n \in S_o} + \sum_{n \in S_r} \), which is a quadratic expression in all \( \alpha \)-parameters.

Up to this point we have assumed the \( \beta \)-parameters, figuring in \( \pi \), to be known. However, as this is not the case, we use a two-step iterative estimation procedure and the steps of this procedure for two-earner households are given below.

- Initiation:
  
  Set \( i = 1 \)

  set \( \beta_1^{(i)} = \ldots = \beta_J^{(i)} = 1 \) and calculate \( \pi_{n}^{(i)} \)

- Step 1:
  
  1. Estimate the system in (33) by means of SUR and obtain \( \hat{\alpha}^{(i)} \) using \( \pi_{n}^{(i)} \)

- Step 2:
  
  1. Using \( \hat{\alpha}^{(i)} \) and estimate \( \beta_1^{(i+1)}, \ldots, \beta_J^{(i+1)} \) by (non-linear) maximum likelihood
  2. calculate \( \pi_{n}^{(i+1)} \)
  3. \( i = i + 1 \)
  4. Stop , if \( \text{abs}(\pi_{n}^{(i)} - \pi_{n}^{(i-1)}) < \frac{1}{1000} \); Goto Step 1 , otherwise.
The non-linear nature of \( \pi_n(\cdot) \) complicates the estimation of the parameters \( \alpha \) in the first estimation step. The advantage of this indirect estimation procedure is that we avoid estimating all model parameters simultaneously, as this would mean that the non-linear \( \pi_n(\cdot) \) function appears in each \( \alpha \)-coefficient. The estimation procedure for one-earner households is equivalent to that of two-earner households, but with the exception that we estimate the system in (34) in Step 1.

4 Data and Estimation Results

4.1 Data

We use the 2003-wave \((J)\) of the British Household Panel Survey \((BHPS)\). This household-based panel began in 1991, and each adult member of the household is interviewed each year. The main objective of the \(BHPS\) is to give insight into the social and economic changes at the individual and household level in the UK.

From the 2003 wave, we use a sub-sample of 1759 couples where household members were interviewed between September 2002 and September 2003. When both spouses are not participating in paid labor or when one of the spouses is self-employed, we do not consider those households in the analysis.

In the \(BHPS\), both spouses are asked to report their participation status. In Table 1, we show the participation status of the women in our sample.

Table 1: Participation Status Women

<table>
<thead>
<tr>
<th>Women</th>
<th>Freq.</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>1496</td>
<td>85.10</td>
</tr>
<tr>
<td>No</td>
<td>262</td>
<td>14.90</td>
</tr>
<tr>
<td>Total</td>
<td>1759</td>
<td>100.00</td>
</tr>
</tbody>
</table>

The reason for showing only the participation status for women and not for men is because there are only 40 households where only the woman works. We will not consider these households. Approximately 15 percent of the women in our sample do not participate in paid labor. In order to have a better understanding of who these persons are, we show the gender-specific descriptive statistics in Table 2. We distinguish between one- and two-earner households and between men and women. The level of education is measured on a 7-
Table 2: Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>Two-earner Households</th>
<th>One-earner Households</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Men</td>
<td>Women</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>Std.Dev.</td>
</tr>
<tr>
<td>Leisure hours per week</td>
<td>118.7</td>
<td>9.5</td>
</tr>
<tr>
<td>Housework hours per week</td>
<td>5.3</td>
<td>4.2</td>
</tr>
<tr>
<td>Job hours per week</td>
<td>43.9</td>
<td>9.0</td>
</tr>
<tr>
<td>Net hourly wage</td>
<td>7.7</td>
<td>3.4</td>
</tr>
<tr>
<td>Age</td>
<td>40.0</td>
<td>10.0</td>
</tr>
<tr>
<td>Education level</td>
<td>3.7</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td>Men</td>
<td>Women</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>Std.Dev.</td>
</tr>
<tr>
<td>Leisure hours per week</td>
<td>118.8</td>
<td>10.6</td>
</tr>
<tr>
<td>Housework hours per week</td>
<td>5.0</td>
<td>4.0</td>
</tr>
<tr>
<td>Job hours per week</td>
<td>44.2</td>
<td>9.9</td>
</tr>
<tr>
<td>Net hourly wage</td>
<td>8.3</td>
<td>4.9</td>
</tr>
<tr>
<td>Age</td>
<td>39.6</td>
<td>10.5</td>
</tr>
<tr>
<td>Education level</td>
<td>3.5</td>
<td>1.8</td>
</tr>
</tbody>
</table>
point scale, where 1 is the lowest possible education level (no study was finished), and 7 is the highest possible education level (higher degree). Men are on average older, higher-educated and have a higher hourly wage than women, which is a common finding in empirical studies. The spouses of two-earner households are higher educated and older than the spouses of one-earner households.

Table 2 also shows information concerning the time use of the spouses. The information on housework time is obtained by asking spouses the following question:

“About how many hours do you spend on housework in an average week, such as time spent on cooking, cleaning, and doing the laundry?”

Clearly, the time spent on housework contains the time spent on cooking, cleaning, and doing the laundry, but the addition such as indicates that spouses can include other housework activities as well, such as child care. Consequently, the distinction between housework and leisure is ambiguous and the empirical definition is left to the respondents themselves.

Women spend more hours on housework and, if they participate in paid labor, they work less hours than men. The paid labor hours of men in one-earner households is about equal to that of men in two-earner households, and the same holds for leisure and housework. Women in one-earner households do not perform paid labor, and as a consequence they spend more hours on leisure and housework.

The total number of housework hours is higher for one-earner households than it is for two-earner households. One explanation is that the spouses of two-earner households earn more income and that this income is (partly) used to outsource some of the household tasks that they dislike. Examples of outsourcing are hiring a cleaner and buying a dishwasher. Another explanation is that the spouses of one-earner households have on average more children, so that there is more housework to do. This can also be the reason why women of one-earner households do not participate in the labor market in the first place.

In Table 3 we show the descriptive statistics that concern the children that are present in the household and the household income. The number of children are divided into four subgroups, so that differences in the number of children between one- and two-earner households can be linked to the children’s age. The average number of children below 12 years old is much higher for one-earner households than it is for two-earner households. On average there are about three times as many children aged below 5 in one-earner households. This confirms the idea that the decision to have children is endogenous with the labor participation choice of women, i.e. women choose not to work when they do not choose to have a child, and visa versa.

The household income of two-earner households is substantially higher than that of one-earner households. The difference in the household labor income is even more pronounced because one-earner households receive about 45 pounds more unearned income than two-earner households. Unfortunately, there is no information available in the data on the amount of money that spouses spend on the outsourcing of household tasks. Therefore we cannot examine whether high income households spend more money on the outsourcing of
Table 3: Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>Two-earner</th>
<th>One-earner</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std.Dev.</td>
</tr>
<tr>
<td>Child present aged between 0-2</td>
<td>0.1</td>
<td>0.3</td>
</tr>
<tr>
<td>Child present aged between 3-4</td>
<td>0.1</td>
<td>0.3</td>
</tr>
<tr>
<td>Child present aged between 5-12</td>
<td>0.4</td>
<td>0.7</td>
</tr>
<tr>
<td>Child present aged &gt;12</td>
<td>0.2</td>
<td>0.4</td>
</tr>
<tr>
<td>Weekly net household income</td>
<td>583.7</td>
<td>214.0</td>
</tr>
<tr>
<td>Weekly net non-labor household income</td>
<td>40.0</td>
<td>71.3</td>
</tr>
</tbody>
</table>

household tasks.

4.2 Estimation Results

Since we do not observe the wage rate of the non-participating women, we predict their market wage rates by making use of a Heckman sample selection model (Heckman, 1979). The intuition behind this model is that persons with more favorable characteristics have a higher probability of participating in paid labor and also have a higher wage rate. If we were to estimate a wage equation using OLS, it can be that the predicted wage rates are, on average, upwards biased due to a censored wage rate. We will show the estimation results of the Heckman model together with a more detailed explanation in Appendix 5B.

The predicted wage rate of the non-participating women ($\tilde{w}_f$) and that of the woman who are participating in paid labor ($w_f$) are presented in Table 4.

Table 4: Wage Rates of Women

<table>
<thead>
<tr>
<th></th>
<th>Freq.</th>
<th>mean</th>
<th>Std.Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{w}_f$</td>
<td>262</td>
<td>5.88</td>
<td>1.22</td>
</tr>
<tr>
<td>$w_f$</td>
<td>1496</td>
<td>6.45</td>
<td>2.89</td>
</tr>
</tbody>
</table>

The average wage rate predicted for the non-participating women is lower than the average wage rate of the participating women. On the basis of the descriptive statistics on the level of education and age in Table 2, this is what we would expect.
By using the information mentioned in Tables 2 up to 4, we can now estimate the model as is explained in Section 3. The estimates of the preference parameters of one- and two-earner households are presented in Table 5.

### Table 5: Estimates of Preference Parameters

<table>
<thead>
<tr>
<th></th>
<th>One-earner</th>
<th>Two-earner</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>t-value</td>
</tr>
<tr>
<td><strong>Men</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Leisure</td>
<td>1.041</td>
<td>97.62</td>
</tr>
<tr>
<td>Housework</td>
<td>0.004</td>
<td>9.08</td>
</tr>
<tr>
<td>Household production ((H))</td>
<td>-0.063</td>
<td>-4.84</td>
</tr>
<tr>
<td>(H) interaction term</td>
<td>0.068</td>
<td>3.79</td>
</tr>
<tr>
<td>Household income</td>
<td>-0.061</td>
<td>-3.05</td>
</tr>
<tr>
<td>Job hours</td>
<td>0.010</td>
<td>0.55</td>
</tr>
<tr>
<td><strong>Women</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Leisure</td>
<td>0.521</td>
<td>46.25</td>
</tr>
<tr>
<td>Housework</td>
<td>-0.021</td>
<td>-11.96</td>
</tr>
<tr>
<td>Household production ((H))</td>
<td>0.136</td>
<td>10.07</td>
</tr>
<tr>
<td>(H) interaction term</td>
<td>-0.045</td>
<td>-2.69</td>
</tr>
<tr>
<td>Household income</td>
<td>0.408</td>
<td>28.38</td>
</tr>
<tr>
<td>Job hours</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>0.975</td>
<td></td>
</tr>
</tbody>
</table>

For two-earner households we find that leisure and the amount of household income are the most important variables in the individual utility function for both spouses.

For one-earner households we cannot identify the woman’s preference with respect to the number of job hours and so we report dots in Table 5 for \(\alpha_{5,f}\). Because the parameter \(\alpha_{5,f}\) for two-earner households is close to zero we can compare the absolute values of the parameters for both household types as well.

For women we find that leisure and household income are the most important variables in their utility
function. However, women in one-earner households value leisure time less and household income more than the women of two-earner households do. These women also value household production positively in their utility function, and this can be explained by the fact that there are, on average, more children present in one-earner households than in two-earner households.

When we compare men in one- and two-earner households, they appear to have different preferences. Although leisure is the most important variable for men in both household types, men in one-earner households value their leisure time even more than men in two-earner households do. The household income enters negatively in the utility function of men, and this effect is the opposite of what we found for men in two-earner households. It implies that for them income and leisure are complementary goods and not substitutes.

The differences between one-earner and two-earner households appear to be consistent with the specialization behavior of the spouses. The one-earner man is the sole provider of the household labor income, and his non-participating wife takes care of the children and performs household tasks. A consequence of this specialization behavior is that each spouse values those activities the most that are scarce for them.

Table 5 also shows the $\gamma$ parameter that represents the marginal productivity of the woman relative to that of the man. The first order conditions that we estimate for one- and two-earner households are not linear in the $\gamma$-parameter. Therefore we estimate $\gamma$ numerically and perform a grid search method on an interval level. We vary $\gamma$ with a width of 0.025 and choose that value of $\gamma$ that gives the highest log-likelihood of the system that is estimated.

The $\gamma$-parameters are 0.925 and 0.975 for two- and one-earner households, respectively. An explanation can be that, on average, the man is somewhat more efficient when spending an additional hour on household tasks, simply because the woman spends more time on household tasks. We note that $\gamma$ can also be different from one for reasons that are non-related to productivity. Therefore, the main reason to include the $\gamma$-parameter into our model is to make the model more flexible by allowing for the fact that the rate of substitution may be different from one.

In Table 6 we present the parameters of the utility weight function.
Table 6: Estimates of the Utility Weight Function

<table>
<thead>
<tr>
<th></th>
<th>One-earner</th>
<th></th>
<th>Two-earner</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>t-value</td>
<td>Estimate</td>
<td>t-value</td>
</tr>
<tr>
<td>Log($w_m$)</td>
<td>0.278</td>
<td>3.63***</td>
<td>0.714</td>
<td>27.18***</td>
</tr>
<tr>
<td>Log($w_f$)</td>
<td>-0.031</td>
<td>-0.30</td>
<td>-0.742</td>
<td>-28.11***</td>
</tr>
<tr>
<td>Log($y_u+1$)</td>
<td>-0.040</td>
<td>-3.28***</td>
<td>0.006</td>
<td>0.95</td>
</tr>
<tr>
<td>Log(#-children 0/2+1)</td>
<td>-0.106</td>
<td>-1.79*</td>
<td>-0.004</td>
<td>-0.09</td>
</tr>
<tr>
<td>Log(#-children 3/4+1)</td>
<td>0.036</td>
<td>0.57</td>
<td>-0.009</td>
<td>-0.22</td>
</tr>
<tr>
<td>Log(#-children 5/11+1)</td>
<td>-0.098</td>
<td>-2.33**</td>
<td>0.007</td>
<td>0.30</td>
</tr>
<tr>
<td>Log(#-children 12/15+1)</td>
<td>-0.110</td>
<td>-2.17**</td>
<td>-0.008</td>
<td>-0.28</td>
</tr>
<tr>
<td>Log(#-children &gt;15+1)</td>
<td>0.032</td>
<td>0.43</td>
<td>0.002</td>
<td>0.04</td>
</tr>
<tr>
<td>Log($age_m$)</td>
<td>0.240</td>
<td>1.76*</td>
<td>-0.008</td>
<td>-0.12</td>
</tr>
<tr>
<td>Log($age_f$)</td>
<td>-0.342</td>
<td>-2.07**</td>
<td>0.011</td>
<td>0.15</td>
</tr>
<tr>
<td>$\pi$</td>
<td>0.468</td>
<td></td>
<td>0.532</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>262</td>
<td></td>
<td>1496</td>
<td></td>
</tr>
<tr>
<td>Log Likelihood</td>
<td>4046.1504</td>
<td></td>
<td>31487.118</td>
<td></td>
</tr>
</tbody>
</table>

Note: * significant at the 10% level, ** significant at the 5% level, *** significant at the 1% level.

The dominant variables in the utility weight distribution for two-earner households are the hourly wage rates. In fact, the only factor that seems to matter is the ratio of the log hourly wages because the wage estimates for man and woman are about equal. This means that an increase of a person's wage rate implies that this person's utility function is weighted more in the household utility function. The impact of an increase in wage rate on the utility weight distribution is about equal for men and women.

The estimation results differ for one-earner households. The man’s wage rate is positive and significant, but the estimate is smaller compared with that of men in two-earner households. Therefore, a wage increase of the man in one-earner households implies that his utility function is weighted more in the household utility function but the impact of a wage increase is less strong than for men in two-earner households.

The predicted wage rate of the woman does not enter the utility weight function significantly. Pollak (2005) reasons that the utility weight distribution is influenced by the spouses’ wage rates as it represents
the potential earnings. This reasoning is not confirmed by our study, at least not for the women in our sample.

For these women, we find that it is not the potential earnings that matter, but that the children variables are important instead. An explanation is that women who do not participate in our sample could find a job but choose not to in order to take care of the children. We find a strong child effect on the utility weight distribution that is to the advantage of women. We also find a small but significant effect of unearned income in the advantage of the woman.

Another difference between one- and two-earner households is that the age variables matters for one-earner households. As spouses become older the utility weight distribution tends to shift in their advantage. The impact of the woman’s age is higher than that of the man’s age. Furthermore, a small but significant effect is found for the unearned income.

In Table 6 we also show the average value of the utility weight function for both household types. For one-earner households, we find that the utility function of women is weighted more in the household utility function than that of men ($\pi = 0.468$), while we find the opposite for two-earner households ($\pi = 0.532$).

5 When Two-earner Households behave as One-earner households

In this section we describe the behavior of two-earner households as if each household maximizes a household utility function considering the labor supply of the woman as a non-zero constant. The household utility function that is maximized with respect to leisure time and housework time is similar to the household utility function in (4), but the time constraint for the woman in two-earner households now becomes $le_f + wh_f = T - \tilde{j}h_f$, so that we can substitute $wh_f = T - \tilde{j}h_f - le_f$. Consequently, the system that is estimated for two-earner households resembles that of the system in (34), where we take into account that women supply a positive amount of labor by considering $\tilde{j}h_f$ as a constant in the woman’s time constraint.

Estimation of this alternative model for two-earner households is informative in two ways. First, we can examine how the estimation results for two-earner households change when we assume that the woman’s paid labor supply may not be optimally chosen. Secondly, we can compare the model estimates of one-earner and two-earner household where the system of first-order conditions is now equivalent for both household types.

Additionally, we consider for all households in the sample that the woman’s labor supply is constant, and estimate the model simultaneously for one- and two-earner households. In doing so, we assume that the preferences of both household types are similar.

In Table 7 we present the estimation results of these two alternative models.
First, we focus on the alternative estimation results for the two-earner households. We find that the preference parameters for men do not deviate much from the estimated preference parameters for two-earner households in Table 5. For women, we observe a preference shift, where the household income is weighted more and leisure is weighted less compared with the two-earner estimates of Table 5. Nevertheless, it is the case that the household income and leisure are still the most important variables in the utility function of both spouses.

When we assume that \( jh_f = \text{constant} \) for all households and estimate the model simultaneously for both household types, we find that the estimates are in-between the estimates for one- and two-earner households.
separately. The estimates obtained for men are more similar to those of the two-earner situation, while the estimates for women are more similar to those of the one-earner situation.

The value of $\gamma$ represents the marginal productivity of the woman relative to that of the man. This value does not vary much over the different alternatives that we estimated and we conclude that the marginal rate of substitution is slightly smaller than 1 for both one- and two-earner households.

The estimation results of the utility weight function are presented in Table 8. The estimation results represented by the One-earner and Two-earner columns are those obtained earlier in Table 6 and we have included these results so that the reader can compare the variation of the estimates over the different alternatives.
Table 8: Alternative Estimates of the Utility Weight Function

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Two-earner</td>
<td>All households</td>
<td>One-earner</td>
<td>Two-earner</td>
</tr>
<tr>
<td></td>
<td>constant $\tilde{J}_f$</td>
<td>constant $J_f$</td>
<td>(from Table 6)</td>
<td>(from Table 6)</td>
</tr>
<tr>
<td></td>
<td>Estimate</td>
<td>$t$-value</td>
<td>Estimate</td>
<td>$t$-value</td>
</tr>
<tr>
<td>$\log(w_m)$</td>
<td>0.858</td>
<td>22.87***</td>
<td>0.530</td>
<td>7.90***</td>
</tr>
<tr>
<td>$\log(w_f)$</td>
<td>-0.605</td>
<td>-20.42***</td>
<td>-0.276</td>
<td>-7.70***</td>
</tr>
<tr>
<td>$\log(y_{u+1})$</td>
<td>-0.076</td>
<td>-9.94***</td>
<td>-0.019</td>
<td>-2.40***</td>
</tr>
<tr>
<td>$\log(#-children \ 0/2+1)$</td>
<td>0.297</td>
<td>6.07***</td>
<td>0.119</td>
<td>2.39**</td>
</tr>
<tr>
<td>$\log(#-children \ 3/4+1)$</td>
<td>0.276</td>
<td>5.51***</td>
<td>0.212</td>
<td>2.60**</td>
</tr>
<tr>
<td>$\log(#-children \ 5/11+1)$</td>
<td>0.136</td>
<td>4.79***</td>
<td>0.015</td>
<td>0.48</td>
</tr>
<tr>
<td>$\log(#-children \ 12/15+1)$</td>
<td>0.120</td>
<td>3.63***</td>
<td>-0.011</td>
<td>-0.39</td>
</tr>
<tr>
<td>$\log(#-children \ &gt;15+1)$</td>
<td>-0.032</td>
<td>-0.56</td>
<td>-0.033</td>
<td>-0.59</td>
</tr>
<tr>
<td>$\log(age_m)$</td>
<td>-0.054</td>
<td>-0.69</td>
<td>0.109</td>
<td>1.14</td>
</tr>
<tr>
<td>$\log(age_f)$</td>
<td>0.090</td>
<td>1.14</td>
<td>-0.161</td>
<td>-1.45</td>
</tr>
<tr>
<td>$\pi$</td>
<td>0.722</td>
<td>0.633</td>
<td>0.468</td>
<td>0.532</td>
</tr>
<tr>
<td>$N$</td>
<td>1496</td>
<td>1758</td>
<td>262</td>
<td>1496</td>
</tr>
<tr>
<td>Log-Likelihood</td>
<td>22737.771</td>
<td>26620.161</td>
<td>4046.150</td>
<td>31487.118</td>
</tr>
</tbody>
</table>

Note: * significant at the 10% level, ** significant at the 5% level, *** significant at the 1% level.
When we estimate the model for all households under the assumption that $jh_f = \text{constant}$, we find that the estimates are between those obtained for two-earner households, where $\widetilde{jh}_f = \text{constant}$, i.e. (1), and those obtained for one-earner households, where $jh_f = 0$, i.e. (3). Therefore, we conclude that one-earner and two-earner households are distinct groups with their own set of preference parameters and their own set of utility weight parameters.

When we estimate the model where $\widetilde{jh}_f$ is considered as a constant for two-earner households, i.e. (1), we find that the utility weight distribution depends on the wage rate of both spouses. Compared with the estimates for two-earner households in Table 6, we find that the wage rate of men becomes somewhat more important, while we find the opposite for the wage rate of women. For the unearned income we now find a small but negative effect on the utility weight distribution.

The most important change in the estimation results is caused by the presence of children in the household. While we found no effect for two-earner households in Table 6, and found that the presence of children was to the advantage of women for one-earner households, we find in (1) that the presence of children is to the advantage of men.

An explanation is that the women in two-earner households are, in general, also the main child care providers in the household. In this case, women are responsible for earning a substantial share in the household income, and the presence of (more) children increases the burden on these working women. Hence, although this situation can be beneficial for the entire household, it is not necessarily beneficial for the woman.

6 Labor Supply Effects

The amount of labor supply for the woman is not necessarily the optimal amount of labor supply if we consider the labor supply of the woman as a constant. In this case, it is interesting to calculate $\frac{\partial U_h}{\partial jh_f}$, so that we can examine how an additional hour of female labor supply would influence the household utility.

Using the appropriate household utility function for both household types, we determine $\frac{\partial U_h}{\partial jh_f}$ and by substituting all the relevant parameter estimates and household characteristics, we may then assess the value of $\frac{\partial U_h}{\partial jh_f}$. We will denote this value by $\tau_h$, and are particularly interested in the sign of that magnitude.

For women of one-earner households this variable $\tau$ represents the marginal non-zero utility of working for the household. When we find that $\tau_h < 0$, this means that an increase of the number of paid labor hours of the female, i.e. the female starts working, leads to a decrease of the household utility level. When we find that $\tau_h > 0$ this means that an increase of paid labor hours leads to an increase of the household utility level. In the latter case it would be beneficial for the household if the woman were to work a positive number of paid labor hours. In a similar way, we can define $\frac{\partial U_f}{\partial jh_f} = \tau_f$ and $\frac{\partial U_m}{\partial jh_f} = \tau_m$. The variables $\tau_f$ and $\tau_m$ represent the effect that an increase of the paid labor hours of the woman has on the utility of, respectively, the female and the male.
If we assume for two-earner households that the amount of labor supply for the woman is not necessarily the optimal amount of labor supply then we can calculate the values $\tau_h$, $\tau_m$ and $\tau_f$ in a similar way. For these two-earner households the variable $\tau$ represents the marginal utility when the woman works slightly more labor hours than she currently does. The interpretation of the signs of $\tau$ is similar for one- and two earner households.

It should be noted that the $\tau$ variables are ordinal variables. The sign of $\tau$ is interpretable, but the absolute value is not interpretable without making further assumptions. So we do not know whether the difference between $\tau = 2$ and $\tau = 3$ is large or non-significant in terms of a deviation from the equilibrium. However, we do know that the amount of labor is optimally chosen whenever we find that $\tau = 0$. In Table 9, we find that the calculated average values of $\tau$ are all significantly different from zero.

Table 9: The Marginal Utility of Labor

<table>
<thead>
<tr>
<th>One-earner</th>
<th>Two-earner</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_h$</td>
<td>+</td>
</tr>
<tr>
<td>$\tau_m$</td>
<td>-</td>
</tr>
<tr>
<td>$\tau_f$</td>
<td>+</td>
</tr>
</tbody>
</table>

*Note: All estimates are significantly different from zero at the 1% level.*

The average values of the $\tau$ variables are significant and positive for two-earner households. An increase of the woman’s labor hours would therefore be beneficial for the man, the woman, and the household. This is an interesting result as a marginal increase in the woman’s labor hours is a Pareto-improvement, i.e. at least one household member benefits from the marginal increase, while no household member is worse-off.

For one-earner households we find that an increase of the number of paid labor hours of the female, i.e. the female starts working, leads to a increase of the household utility level and her own utility level, but it will negatively affect the utility level of the man. Interestingly, we observe that the observed labor supply of women is Pareto-efficient, although the woman herself benefits from working.

7 Conclusion

In this study we examined how spouses allocate their time to paid labor, leisure and housework and allowed for the possibility that one of the partners in the household is not active in a paid job. For this purpose we used the BHPS data source. For one-earner households we only examined the situation where the woman does not participate in paid labor because usually the male is the only income earner in one-earner households.
We estimated a collective time allocation model separately for one- and two-earner households. The behavior of two-earner households is described as if the spouses maximize a household utility function and the behavior of one-earner households is described as if the spouses maximize a household utility function conditional on the zero job-hour choice of the woman.

We find that leisure and household income are the most important variables in the individual utility functions. The non-participating women value leisure somewhat less and household income somewhat more than the women in two-earner households do. The same holds for the total amount of household production and this may be explained by the fact that there are, on average, more children present in one-earner households than in two-earner households. Men in one-earner households find leisure much more important than men in two-earner households do, and value the household income negatively in the utility function. The latter effect is the opposite of what we found for men in two-earner households. The differences between one-earner and two-earner households may reflect the specialization behavior of the spouses.

With respect to the utility weight distribution, we find that the utility function of non-participating women is weighted more in the household utility function than that of men, while we find the opposite for two-earner households. An increase of a person’s wage rate implies that this person’s utility function is weighted more in the household utility function, but only if this person is participating in paid labor. For the non-participating woman, we observe that the presence of children improves her bargaining position but her wage rate, which is her potential wage rate that she can earn when she works, is not significant. It seems that women obtain their bargaining power through their wages when they perform paid labor, and through their children when they decide not to work.

We also described the behavior of two-earner families as if the spouses maximize a household utility function, conditional on the zero job-hour choice of the woman. In this alternative model, the labor supply of the woman may be non-optimal, as was the case for one-earner households. On the basis of the estimation results we conclude that one-earner and two-earner households are distinct groups with their own set of preference parameters and their own set of utility weight parameters.

For two-earner households the men’s preference parameters for the alternative estimation model remain rather similar to the earlier two-earner estimates. For women, on the other hand, we observe a shift in the preference parameters, where the household income is valued more and leisure is valued less compared with the two-earner estimates obtained earlier.

Concerning the utility weight distribution, the most important change is caused by the presence of children in the household. While we found no effect for two-earner households, and found a child effect to the advantage of the woman for one-earner households, we found in the alternative model that the child effect is to the advantage of the man. An explanation is that the women in two-earner households are, in general, also the main child care providers in the household. In this case, women are responsible for earning
a substantial share of the household income, and the presence of (more) children increases the burden on these working women. Although this situation can be beneficial for the entire household, it is not necessarily beneficial for the woman.

Finally, we evaluated how an additional hour of female labor supply would influence the utility level of the household, the man and the woman, when we assume that the labor supply of women may be non-optimal. We note that the labor supply of women may be non-optimal for both one-earner and two-earner households according to the alternative definition. For two-earner households we find that an increase of the woman’s labor hours would be a Pareto improvement because it would benefit the man, the woman, and consequently the entire household. For one-earner households, we find that the household utility and the utility level of the woman would increase if the woman decides to supply a positive amount of paid labor. This increase in labor supply would lower the utility level of the man, and this is interesting because it means that the zero job-hour choice of the woman is actually Pareto-efficient, even though the woman herself would benefit from a positive supply of paid labor.

**Appendix 5.A**

This Appendix shows the system of first order conditions, where the system is written in a way that each preference parameter in each equation had its own coefficient:

\[
\frac{\partial U_h}{\partial l e_m} = \pi [\alpha_{m,2} x_{2,m,2} + \alpha_{m,3} x_{2,m,3} + \alpha_{m,5} x_{2,m,5} + \alpha_{m,6} x_{2,m,6} + \alpha_{m,5} x_{2,m,5}] + (1 - \pi) [\alpha_{f,2} x_{2,f,2} + \alpha_{f,3} x_{2,f,3} + \alpha_{f,4} x_{2,f,4} + \alpha_{f,5} x_{2,f,5}]
\]

\[
\frac{\partial U_h}{\partial w h_m} = \pi [\alpha_{m,2} x_{2,m,2} + \alpha_{m,3} x_{2,m,3} + \alpha_{m,5} x_{2,m,5} + \alpha_{m,6} x_{2,m,6} + \alpha_{m,5} x_{2,m,5}] + (1 - \pi) [\alpha_{f,2} x_{2,f,2} + \alpha_{f,3} x_{2,f,3} + \alpha_{f,4} x_{2,f,4} + \alpha_{f,5} x_{2,f,5}]
\]

\[
\frac{\partial U_h}{\partial l e_f} = \pi [\alpha_{f,2} x_{2,f,2} + \alpha_{f,3} x_{2,f,3} + \alpha_{f,4} x_{2,f,4} + \alpha_{f,5} x_{2,f,5}]
\]

\[
\frac{\partial U_h}{\partial w h_f} = \pi [\alpha_{f,2} x_{2,f,2} + \alpha_{f,3} x_{2,f,3} + \alpha_{f,4} x_{2,f,4} + \alpha_{f,5} x_{2,f,5}]
\]

**Appendix 5.B**

We estimate the probability that a woman participates in paid labor based on characteristics that influence the hourly wage rate (i.e. equation 37). On the basis of these estimates we obtain the inverse Mills ratio and include this variable in the wage regression. The inverse mills ratio corrects for the fact that we do not observe the wage rate for the non-participating women, a wage rate that is on average upwards biased due to a censored dependent variable.

More formally the estimation model is

\[
\log(w) = V'v + \eta_i
\]

A person is in the labor force if:
\[ S' \theta + \eta_2 > 0 \]  

Furthermore, it is assumed that

\[ \eta_1 \sim N(0, \sigma) \]
\[ \eta_2 \sim N(0, 1) \]
\[ \text{corr}(\eta_1, \eta_2) = \rho \]

The explanatory variables that are in the wage regression are represented by \( V \). The variables that determine whether a woman is selection in paid labor is represented by \( S \). The identification in this study relies exclusively on the model and the normality assumption concerning the two error terms being correct. Therefore, the explanatory variables that appear in \( V \) also appear in \( S \).\(^6\) The characteristics that are used as explanatory variables are age, the number of children between certain age levels and the education level. Of all variables we took the logarithm and added one when necessary.

The estimation results of equation (36) and (37) are shown in Table 10.

\(^6\)We used information on whether there has been change in marital status over the past 4 years, but this variable did not significantly enter the selection equation.
Table 10: Heckman Selection Model

Estimates of the Selection Equation

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log(#-children 0/2+1)</td>
<td>-1.238***</td>
<td>-7.94</td>
</tr>
<tr>
<td>Log(#-children 3/4+1)</td>
<td>-1.014***</td>
<td>-6.61</td>
</tr>
<tr>
<td>Log(#-children 5/11+1)</td>
<td>-0.659***</td>
<td>-7.09</td>
</tr>
<tr>
<td>Log(#-children 12/15+1)</td>
<td>-0.116</td>
<td>-0.88</td>
</tr>
<tr>
<td>Log(#-children &gt;15+1)</td>
<td>0.253</td>
<td>-1.01</td>
</tr>
<tr>
<td>Log(education level)</td>
<td>0.424***</td>
<td>6.21</td>
</tr>
<tr>
<td>Log(age level)</td>
<td>0.160</td>
<td>-0.94</td>
</tr>
<tr>
<td>constant</td>
<td>-1.064**</td>
<td>2.55</td>
</tr>
</tbody>
</table>

Estimates of the Wage Equation

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log(#-children 0/2+1)</td>
<td>-0.056</td>
<td>-0.41</td>
</tr>
<tr>
<td>Log(#-children 3/4+1)</td>
<td>0.077</td>
<td>0.63</td>
</tr>
<tr>
<td>Log(#-children 5/11+1)</td>
<td>-0.046</td>
<td>-0.68</td>
</tr>
<tr>
<td>Log(#-children 12/15+1)</td>
<td>-0.082**</td>
<td>-2.23</td>
</tr>
<tr>
<td>Log(#-children &gt;15+1)</td>
<td>0.003</td>
<td>0.04</td>
</tr>
<tr>
<td>Log(education level)</td>
<td>0.362***</td>
<td>8.45</td>
</tr>
<tr>
<td>Log(age level)</td>
<td>0.277***</td>
<td>6.22</td>
</tr>
<tr>
<td>constant</td>
<td>0.299*</td>
<td>1.72</td>
</tr>
<tr>
<td>mills ratio</td>
<td>0.325</td>
<td>1.15</td>
</tr>
</tbody>
</table>

N                          3486
Censored observations      262
Uncensored observations    1496
Wald chi2(16)              454.48

Note: * significant at the 10% level, ** significant at the 5 % level, *** significant at the 1 % level.

The probability that a woman will work is increasing in her education level and decreasing in the number of children, which is as expected. The wage rate of two earner households are mainly influenced by age and education level. Also having children between 12 and 15 years old explains some of the variation in log \( w_f \), but this effect is small compared to the other effects that are found. The Inverse Mills ratio that corrects for selection is not significant.

On the basis of the wage equation estimates we can make an out-of-sample prediction and obtain \( E(w_i | \text{not-working}) \).
References


Browning, M. and M. Gørtz (2005), Spending time and money within the household. working Paper, Institute of Economics, Copenhagen.


